Adaptive Stabilization of Time-varying Discrete-time Linear Systems*

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Abstract—We develop an adaptive control technique for the regulation of a class of linear, discrete-time, time-varying system. The only a priori knowledge required is a bound of the varying component of the parameters. The result is concerned with global behaviour.

1. Introduction
This paper deals with adaptive control techniques implemented for the regulation of linear time-varying systems with unknown parameters. Different authors have tackled the problem of adaptive stabilization of discrete time varying systems. Wang and Jl Jung (1989) studied a single-input single-output case (SISO) with disturbances and obtained bounded output signals. Ydstie (1989) stabilizes systems with additive perturbations. In his work the perturbations represent the mismatch that is incurred when the linear part of the model is applied to the system to be controlled. His result, although it is very general, is based on the knowledge of a compact region that contains the true parameters. Bahanasawi and Mahmoud (1989) developed a nonlinear feedback control strategy in order to stabilize a class of linear discrete time systems with perturbed parameters under certain hypotheses about the disturbances and the fixed components of the parameters. Ilchman (1991) analyzed the case of a first-order SISO system, and proposed a control law that is robust with respect to small time variations of the parameters. He developed a simple control strategy making use of bounds of the feedback gain and of the varying components of the parameters. Meyn and Brown (1993) combined a simple model reference adaptive control law with the Kalman filter. They showed stability of the input-output process. Su Juing-Huei and Fong I-Kong (1993) found bounds for the varying component parameters of a linear time stable system to ensure asymptotically stability. Eslami (1993) constructed the largest set of parameter variations for a stable convergent matrix to remain stable.

In this work we consider a class of partially known, time-varying, discrete-time, not necessarily stable systems. A very simple scheme based on the projection algorithm (Goodwin and Sin, 1984) for linear, time-invariant, discrete-time systems is presented. The objective is to solve the regulation problem for this class of time-varying systems with unknown parameters. We consider that the parameters have a constant component plus a varying one. We obtain a control law with the Kalman filter. They showed stability of the input-output process. Su Juing-Huei and Fong I-Kong (1993) found bounds for the varying component parameters.

2. Description of the system
We consider a linear, time-variant, discrete-time system of order n, of the form
\[ y_{k+1} = (A_k + A_0)Y_k + A_0Y_{k-1} + \ldots + (a_{k+1} + a_1)Y_k + (a_0 + a_1)Y_{k-1} + \ldots + a_1Y_{k-n+1} + U_k. \]

We define
\[ Y_k = [y_k, y_{k-1}, \ldots, y_{k-(n-1)}], \]
\[ A_k = [a_0, a_1, \ldots, a_{k-(n-1)}], \]
\[ A_0 = [a_0, a_1, \ldots, a_{n-1}], \]

where \( A \in \mathbb{R}^n \) is the vector containing the fixed part of the parameters of the system and \( A_0 \) represents the varying component. A bound on the \( \| A_k \|_2 \leq \alpha < 1 \).

The system can be expressed as
\[ y_{k+1} = (A + A_0)Y_k + U_k. \]

Our objective is to solve the stabilization problem. To do this, we propose a slight modification of the projection algorithm, a scalar parameter estimator for time-invariant linear systems (Goodwin and Sin, 1984). We generate a sequence of estimators \( \hat{A}_k \) of A that is updated by
\[ \hat{A}_{k+1} - \hat{A}_k + m_k \frac{y_{k+1}Y_k}{\gamma + \| Y_k \|_2^2}, \]

where \( \gamma \) and \( \beta \) positive real constants. We calculate the control without considering the varying component \( a_1 \):
\[ u_k = -\hat{A}_k Y_k. \]

We ensure that, for every value of \( \alpha \), there exists values of \( \beta \) and \( m \), both depending only on \( \alpha \), such that the resulting closed-loop system tends asymptotically to zero for every choice of \( \gamma > 0 \).

Proposition 2.1. For the system (2), with initial data \( Y_0 \) and \( \hat{A}_0 \), choosing \( 1 > \beta^2 > \alpha \) and defining \( m = \frac{1}{2}(1 - \alpha \beta^2) \), the control law (3) (5) stabilizes the system.

Proof. Let \( A_k = \hat{A}_k - \hat{A}_0 \). The closed-loop system (2), (3) becomes:
\[ y_{k+1} = A_k Y_k + A_0 Y_{k}, \]
and (3) becomes

\[ A_{k+1} = A_k - m_k \frac{y_{k+1} y_k^2}{\gamma + \|Y_k\|^2}, \]  

(7)

\[ m_k = \begin{cases} 0 & \text{for } |y_{k+1}| = \beta \|Y_k\|, \\ m & \text{otherwise.} \end{cases} \]  

(8)

Note that the choice of \( \beta \) ensures that \( m_k \geq 0 \). The sequence \( A_k \) satisfies

\[ \|A_{k+1}\|^2 \leq \|A_k\|^2 - 2m_k \frac{y_{k+1} A_k y_k}{\gamma + \|Y_k\|^2} + m_k^2 \frac{y_{k+1}^2 \|Y_k\|^2}{(\gamma + \|Y_k\|^2)^2}. \]  

(9)

Replacing (6) in (9) and noting that for every \( \gamma > 0, \|Y_k\|^2/(\gamma + \|Y_k\|^2) \leq 1 \), we obtain the following inequality:

\[ \|A_{k+1}\|^2 \leq \|A_k\|^2 - 2m_k \frac{y_{k+1} y_k}{\gamma + \|Y_k\|^2} + m_k \frac{y_{k+1}^2 \|Y_k\|^2}{\gamma + \|Y_k\|^2}. \]  

(10)

For every \( \xi > 0 \), we have:

\[ 2y_{k+1}(A_k y_k) \leq \frac{y_{k+1}^2}{\xi}. \]  

(11)

\[ \Delta \leq \|Y_k\|^2 < \eta \|Y_k\|^2, \text{ and } m_k = 0 \text{ if and only if } \|Y_k\|^2 < \eta \|Y_k\|^2, \text{ applying the Cauchy–Schwarz inequality to (11), we obtain} \]

\[ \|A_{k+1}\|^2 - \|A_k\|^2 \leq -\theta_0 \frac{y_{k+1}^2}{\gamma + \|Y_k\|^2}, \]  

(12)

where

\[ \theta_0 = \begin{cases} \theta & \text{for } |y_{k+1}| > \beta \|Y_k\|, \\ 0 & \text{otherwise}, \end{cases} \]  

with

\[ \theta = \frac{1}{4} \left( 1 - \frac{n\alpha^2}{\beta^2} \right). \]

Summing both sides of (13), we obtain

\[ \|A_0\|^2 \geq \|A_0\|^2 - \|A_{N+1}\|^2 \geq \sum_{k=0}^{\infty} \theta_k \frac{y_{k+1}^2}{\gamma + \|Y_k\|^2}. \]  

(14)

The choice of \( \beta \) and \( \theta \) ensures that the general term on the right-hand side of (14) is always positive, and we can conclude that

\[ \lim_{k \to \infty} \theta_k \frac{y_{k+1}^2}{\gamma + \|Y_k\|^2} = 0, \]

which means that \( \forall \xi > 0 \), there exists \( k_0 \) such that if \( k \geq k_0 \), then \( \theta_k \frac{y_{k+1}^2}{\gamma + \|Y_k\|^2} < \xi \). From this assertion, we shall conclude the stabilization of the system.

We analyze the two possibilities that can arise.

In the case \( \theta_k = 0 \), we arrive at

\[ \theta - \frac{\beta^2}{\gamma + \|Y_k\|^2} < \xi, \]

from which it follows that

\[ (\theta \beta^2 - \xi n) \|Y_k\|^2 < \xi y, \]

and hence, for \( \xi < \theta \gamma n \),

\[ \|Y_k\|^2 < \frac{\xi y}{\beta^2 \theta - \xi n}. \]  

Fig. 1. Evolution of the output of a second-order system.

If \( \theta_k = 0 \), the output decreases strictly each \( n \) iterations, and in the meantime it remains bounded:

\[ |y_{k+1}| < \beta \|Y_k\| \text{ for } j < n, \]

\[ |y_{k+1}| < \beta^2 \|Y_k\| \text{ for } n < j < 2n. \]

The result of the proposition follows.

When the parameters of the system are actually invariant, regulation is also achieved.

Figure 1 illustrates the evolution of the output of a second-order case for \( \alpha = 0.6, \beta = 0.9, \gamma_0 = 2 \) and \( \alpha, = 2 \), with initial conditions \( y_0 = 2, y_1 = -2, \gamma_0 = 1.5 \) and \( \beta_0 = 1 \). The varying component of the parameters is random in both cases.

Figure 2 shows the evolution of the output of a first-order system for \( \alpha = 3, \gamma_0 = 2, \alpha = 0.6, \beta = 0.9 \) and \( \alpha_0 = 1 \) when the varying component is sinusoidal.

Remark 2.1. There exists a range of values where we can choose \( m \) and \( \xi \) in the proposition to achieve our control objective:

\[ 0 < m < \frac{1}{4} - \frac{\xi n \alpha^2}{\beta^2} \text{ and } \xi < \frac{\beta^2 - \sqrt{\beta^4 - \beta^2 \alpha^2}}{\alpha^2} < \frac{\beta^4 - \beta^2 \alpha}{\alpha^2}. \]

Remark 2.2. The convergence of the output of the system

Fig. 2. Evolution of the output of a first-order system.
towards zero is exponentially fast when we know the exact value of $\alpha$ and we use it as the initial value $\tilde{a}_0$:

$$|y_{k+1}| < \beta^{k+1} ||Y_k||_\infty.$$

Figure 3 shows this situation for $a = -3$, $\alpha = 0.4$, $\beta = 0.8$ and $y_0 = 4.5$; in this case $\alpha_k$ is random.

The closer we choose $A_0$ to $A$, the better is the performance of the algorithm. Figures 4 and 5 show the evolution of the output of the closed-loop system for $a_0 = -1$, $y_0 = 4$, $\alpha = 0.5$, $\beta = 0.9$ and $\alpha_k$ random with initial conditions $\tilde{a}_0^0 = 1$ and $\tilde{a}_0^0 = -0.9$ respectively.

3. Conclusions

An algorithm to stabilize the output of a linear time-varying, discrete-time system has been developed. Little a priori knowledge of the system is required to implement it. We only need a bound on the varying component of the parameter to ensure convergence to zero from any initial condition, $Y_0, \tilde{A}_0$. Additional information about the parameter values can be used to obtain better results, in the sense of faster convergence, as the simulations show. The scheme applies to linear time-varying systems of any order. These results could probably be extended to systems subject to different kinds of perturbations.

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References


