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Four-terminal resistance of an interacting quantum wire with weakly invasive contacts

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Abstract

We analyze the behavior of the four-terminal resistance, relative to the two-terminal resistance of an interacting quantum wire with an impurity, taking into account the invasiveness of the voltage probes. We consider a one-dimensional Luttinger model of spinless fermions for the wire. We treat the coupling to the voltage probes perturbatively, within the framework of non-equilibrium Green function techniques. Our investigation unveils the combined effect of impurities, electron–electron interactions and invasiveness of the probes on the possible occurrence of negative resistance.

(Some figures may appear in colour only in the online journal)

1. Introduction

Quantum transport in novel materials is one of the most active areas of present research in condensed matter physics [1]. The problems that arise are especially interesting in one-dimensional (1D) devices such as quantum wires and carbon nanotubes. In these cases the effect of electron–electron (e–e) interactions is crucial, leading to the so called Luttinger liquid (LL) behavior [2], characterized by correlation functions which decay with interaction-dependent exponents [3] and a power law in the tunneling $I$–$V$ characteristic curve [4].

The actual nature of the resistance in a mesoscopic device has been a central issue since the first milestones in the theory of quantum transport. Landauer proposed the famous setup to study quantum transport where a mesoscopic sample is placed between two reservoirs at different chemical potentials [5]. Then, Büttiker [6], in agreement with experiment [7], showed the fundamental relation $G = nG_0$ for the two-terminal conductance of a non-interacting quantum wire, $n$ being the number of transverse channels and $G_0 = e^2/h$ being the universal conductance quantum. The remarkable consequence of this simple law is the fact that a purely non-interacting electronic system without any kind of inelastic scattering mechanism has a sizable resistance, which for a single channel device is as large as $G_0^{-1} \simeq 13$ kΩ. This resistive behavior is due to the coupling between the system and the reservoirs through which the driving voltage is applied. For this reason, this quantity is identified as the contact resistance of the ideal non-interacting setup. The mesoscopic community then became motivated toward the definition of an alternative physical concept to describe the resistive behavior of the sample, free from the effects of the contact resistance. In another pioneering work [8], a gedanken setup was proposed in order to sense the local voltage and the temperature. The main idea is to consider the mesoscopic system locally coupled to voltage probes or thermometers, represented by means of particle reservoirs. The latter have chemical potentials or temperatures that satisfy the condition of local electrochemical or thermal equilibrium with the mesoscopic system, which implies that the chemical potentials and temperatures of these systems are adjusted in order to get vanishing electronic and heat flows through the contacts to the central device. For the case of two voltage probes connected along the sample, as in the sketch of figure 1, the voltage drop corresponding to the chemical potential difference $(\mu_1 - \mu_2)/e$ defines the four-terminal resistance

$$R_{4t} = \frac{\mu_1 - \mu_2}{eI},$$

(1)
where \( I \) is the current flowing through the setup. This scheme to define the four-terminal resistance was later implemented in the framework of scattering matrix theory for multiterminal setups [9] in wires of non-interacting electrons with a single [10, 11] and many [12] impurities. In [10, 11] it is clarified that the inference of \( R_{4t} \) from a calculation based on a two-terminal geometry and the original Landauer formula [5] may not always be correct, which stresses the importance of considering a genuine four-terminal setup to properly evaluate this quantity.

Among other interesting features, for non-interacting systems it was predicted that negative four-terminal resistances are possible [9–13]. This is a consequence of the coherent nature of the electronic propagation along a sample where only elastic scattering processes with barriers or impurities can take place. These negative (longitudinal) resistances in ballistic wires were first measured in the late 1980s [14]. More recently, this effect was also observed in semiconducting structures [15]. A bit later, the behavior of \( R_{4t} \) was experimentally studied in carbon nanotubes [16]. In this case, a negative value of this resistance was also observed within the low temperature regime. Let us mention that the four-terminal resistance, in the context of uncorrelated fermions, has also been extended to the case of time-dependent voltage probes, leading to the concept of four-terminal impedance [17]. It is widely accepted that the Luttinger model of interacting electrons in 1D is able to capture the main features observed in transport experiments on carbon nanotubes [18, 19]. In particular, the power law behavior of the tunneling current as a function of the applied voltage and/or temperature predicted from Luttinger liquid theory has been experimentally observed in these systems. Regarding the behavior of \( R_{4t} \) evaluated from a multiterminal setup in Luttinger liquids, the literature is restricted to [20]. Previous estimates for this quantity were made on the basis of an interpretation of the Landauer formula in a two-terminal setup [21]. This is due to the fact that quantum transport in multiterminal Luttinger liquids or models of interacting electrons is, in general, a rather challenging problem from the technical point of view. Besides [20], genuine multiterminal systems have been considered in Y-type geometries [22, 23], within linear response in the voltage and Hartree–Fock approximation of the interaction, respectively, as well as in the study of the tunneling current of a quantum wire in the Fabry–Perot regime [24]. There are also some recent works on the effect of wires that are capacitively coupled to an additional reservoir [25, 26].

In [20] we considered the setup of figure 1, where an infinite Luttinger wire with a single impurity, through which a current \( I \) flows as a response to an applied voltage \( V \), is connected at two points to voltage probes. Following the procedure of previous works for non-interacting electrons, we considered [8, 10–13] non-invasive contacts between the wire and the voltage probes. We showed that the voltage profile displays Friedel-like oscillations, as in the case of non-interacting electrons [10, 11, 13], but modulated by an envelope displaying a power law behavior as a function of the applied voltage or temperature, with an exponent depending on the electron–electron interaction strength. However, it is known that in the opposite limit of strong enough coupling between the mesoscopic device and the probes, inelastic scattering events and classical resistive behavior take place [27]. Moreover, ideal non-invasive probes cannot be easily realized in experimental situations. For this reason, the aim of the present study is to go a step beyond the assumption of non-invasive probes by considering probes that, while still weakly coupled to the sample, introduce decoherence through inelastic scattering processes, as well as inter-probe interference effects. Among other interesting questions, our goal is to answer whether features in the behavior of the four-terminal resistance determined by non-invasive probes, like Friedel oscillations, or a negative value of the resistance, are still possible when the coupling of the probes becomes invasive. We address these issues in the framework of non-equilibrium Green functions and a perturbative treatment in the coupling to the probes.

The work is organized as follows. In section 2, we present the model and the theoretical treatment to evaluate \( R_{4t} \). In sections 3 and 4, we present results for the clean wire and the wire with an impurity, respectively. Finally, we present a summary and conclusions in section 5. Some technical details are presented in an appendix.

2. Theoretical treatment

2.1. Model

We consider the setup of figure 1. As in [20], we use the following action to describe the full system:

\[
S = S_{\text{wire}} + S_{\text{imp}} + S_{\text{res}} + S_{\text{cont}},
\]

where \( S_{\text{wire}} \) corresponds to an infinite-length Luttinger wire and reads

\[
S_{\text{wire}} = \int \text{d}x \, \text{d}r \left\{ \psi_r^\dagger \left[ i(\partial_t - \partial_x) - \mu_r \right] \psi_r + \psi_l^\dagger \left[ i(\partial_t + \partial_x) - \mu_l \right] \psi_l - \mu_r \psi_r^\dagger \psi_r - g \left[ \psi_r^\dagger \psi_r + \psi_l^\dagger \psi_l \right]^2 \right\},
\]

with the first two terms corresponding to free spinless left and right movers respectively and \( g \) is the e–e interaction in the forward channel. We use units where \( \hbar = k_B = 1 \). We also take the Fermi velocity of the electrons \( v_F = 1 \) and the electronic charge \( e = 1 \). The two chemical potentials \( \mu_l = \mu - V/2 \) and
in the notation, it is convenient to carry out the following gauge transformations: \( \psi_{1l_j}(x) \rightarrow \psi_{1l_j}(x)e^{i\delta y_j}, \quad \chi_{1l_j}(y_j) \rightarrow e^{i\delta y_j} \chi_{1l_j}(y_j) \). The Dyson equation for the retarded function reads

\[
-\text{i}\partial_t + k^0_\beta \lesssim + i\partial_x \chi_{i\beta}(x, x'; t, t')
- \lambda_B \delta(x' - x) \delta(t - t')\delta_{\alpha\beta}
+ \sum_j w_j G_{\alpha\beta}(x, y_j; t, t')\delta(x' - x).
\]

(10)

where the upper and lower signs correspond to \( \beta = l, r \) and \( \beta_j = l_j, r_j, \) respectively, and \( \bar{\ell} = r, \bar{r} = l \). Thus, equation (11) can be expressed as follows:

\[
G_{\alpha\beta}(x, y_j; t, t') = w_j \sum_{\beta} G_{\alpha\beta}(x, y_j; t, t')
\times \delta(x - y_j)\delta(x' - x)\delta(t - t')\delta_{\beta\beta_j}.
\]

(13)

Substituting the latter equation into equation (11) and defining

\[
\Sigma_{\alpha\beta}(x, x'; t, t') =
\sum_{j=1,2,\beta_j} \delta(x - x_j)\delta(x' - x_j)\delta(t - t')\delta_{\beta\beta_j}.
\]

(14)

The lesser Green function entering the expression for the currents \( I_j \) can be obtained by means of Langreth rules from (11) [28], according to which given \( C^R(t, t') = \int dt'' A^R(t, t'')B^R(t'', t') \) then \( C^\alpha(t, t') = \int dt''[A^R(t, t'')B^\alpha(t'', t')] + A^\alpha(t, t')B^R(t'') \), the advanced function being \( [B^A]^T \lesssim \). Thus

\[
G_{\alpha\beta}(x_j, y_j; t, t') = w_j \int dt''[G_{\alpha\beta}(x_j, y_j; t, t'')g_{\beta\beta_j}(y_j, y_j; t'', t', t')
+ G_{\alpha\beta}(x_j, y_j; t, t'')g_{\beta\beta_j}(y_j, y_j; t', t'').
\]

(16)
where $g^A = [g^R]^\dagger$ is the advanced Green function of the uncoupled reservoir.

So far all the equations are exact. The crucial step to obtain the exact Green function by solving Dyson equations is the evaluation of $\Sigma^{\text{int}}$, which corresponds to the fully dressed skeleton diagram for the self-energy associated to the electron–electron interaction, also taking into account the coupling to the two additional reservoirs as well as the backward impurity. We now introduce the following approximation for the limit of weak coupling to the reservoirs and the impurity such that $w_j \ll g$ and $\lambda_B \ll g$:

$$\Sigma^{\text{int}}(x', t, t') \simeq \Sigma^{\text{Lutt}}(x', t, t'),$$

(17)

where

$$\Sigma^{\text{Lutt}}(x', t, t') = (G^{R,\text{Lutt}}(x, x', t, t'))^{-1} \left[ -i\delta_{t, t'} \mp ki \right] \delta_{\beta\beta},$$

(18)

is the self-energy of the infinite Luttinger wire without impurity and uncoupled from the reservoirs, while $G^{R,\text{Lutt}}(x, x', t, t')$ is the ensuing retarded Green function. The approximation (17) implies the evaluation of the self-energy associated to $e-e$ interaction by neglecting vertex corrections due to the escape to the reservoirs and due to the scattering with the impurity. This approximation is adequate only in the limit of small $w_j$ and $\lambda_B$.

Under this approximation in the $e-e$ self-energy and performing a Fourier transform with respect to $t - t'$, equation (15) can be expressed as follows:

$$G^{R,\text{Lutt}}_{\alpha\beta}(x, x'; \omega) = G^{R,\text{Lutt}}_{\alpha\beta}(x, x'; \omega) + \sum_j G^{R}_{\alpha\alpha}(x, x_j, \omega) \Sigma^{\text{res}}_{\alpha\beta}(x_j, x_j'; \omega) G^{R,\text{Lutt}}_{\beta\beta}(x_j, x_j'; \omega) + G^{R}_{\alpha\beta}(x, x_0, \omega) \lambda_B G^{R,\text{Lutt}}_{\beta\beta}(x_0, x_0'; \omega).$$

(19)

This equation allows for the evaluation of the retarded Green function. In what follows, we solve it at the lowest order in the back-scattering term $\lambda_B$ and up to $O(w_j^4)$ in the coupling to the voltage probes. We recall that ideal non-invasive probes correspond to keeping only up to $O(w_j)$ in the equation leads to the exact retarded Green function of the problem.

2.3. Currents

Substituting equation (16) into the definition of the current (7), we obtain the following exact equation for the current flowing through the contact between the $j$th reservoir and the wire:

$$I_j = -2 \sum_{\alpha, \beta} \left\{ w_j^2 \int_{-\infty}^{\infty} \frac{\text{d}\omega}{2\pi} \left[ G^{R,\text{Lutt}}_{\alpha\alpha}(x_j, x_j, \omega) g^{A}_{\beta\beta}(x_j, x_j, \omega) + G^{R}_{\alpha\alpha}(x_j, x_j, \omega) g^{A}_{\beta\beta}(x_j, x_j, \omega) \right] \right\} .$$

(20)

Making use of the assumption of weak coupling between the probes and the wire and weak amplitude in the back-scattering term induced by the impurity, we evaluate the Green functions $G_{\alpha\alpha}^R(x_j, x_j, \omega)$ and $G_{\alpha\alpha}^R(x_j, x_j, \omega)$ perturbatively up to $O(w_j^4)$ and $O(\lambda_B)$. Concretely, this implies solving (19) as

$$G^{R}_{\alpha\alpha}(x, x'; \omega) \simeq G^{R,\text{Lutt}}_{\alpha\alpha}(x, x'; \omega) + \sum_j G^{R,\text{Lutt}}_{\alpha\alpha}(x, x_j, \omega) \Sigma^{\text{res}}_{\alpha\beta}(x_j, x_j'; \omega) G^{R,\text{Lutt}}_{\beta\beta}(x_j, x_j'; \omega) + G^{R}_{\alpha\alpha}(x, x_0, \omega) \lambda_B G^{R,\text{Lutt}}_{\beta\beta}(x_0, x_0'; \omega),$$

(21)

while the lesser counterpart can be derived from (21) by recourse to Langreth rules (see the text above equation (16)).

The explicit expression for $G^{R,\text{Lutt}}_{\alpha\alpha}(x, x_j, \omega)$ is given in the appendix. After some algebra, the currents through the contacts can be expressed as follows:

$$I_j = I_j^{(1)} + I_j^{(2)},$$

(22)

where $I_j^{(1)} \propto |w_j|^2$, $I_j^{(2)} \propto |w_j|^4$, with

$$I_j^{(1)} = 2w_j^2 \sum_{\alpha=1,2} \int_{-\infty}^{\infty} \frac{\text{d}\omega}{2\pi} \left\{ G^{R,\text{Lutt}}_{\alpha\alpha}(x_j, x_j, \omega) g^{R}_{\alpha\beta}(x_j, x_j, \omega) - G^{R,\text{Lutt}}_{\alpha\alpha}(x_j, x_j, \omega) g^{R}_{\alpha\alpha}(x_j, x_j, \omega) \right\},$$

(23)

$$I_j^{(2)} = 4w_j^2 \sum_{\alpha=1,2} \sum_{\beta=0,1} \int_{-\infty}^{\infty} \frac{\text{d}\omega}{2\pi} \left\{ \left[ g^{R}_{\alpha\beta}(x_j, x_j, \omega) g^{A}_{\alpha\beta}(x_j, x_j, \omega) - g^{R}_{\alpha\alpha}(x_j, x_j, \omega) g^{A}_{\alpha\beta}(x_j, x_j, \omega) \right] \times \left[ G^{R,\text{Lutt}}_{\alpha\alpha}(x_j, x_j, \omega) g^{A}_{\alpha\beta}(x_j, x_j, \omega) - G^{R,\text{Lutt}}_{\alpha\alpha}(x_j, x_j, \omega) g^{A}_{\alpha\beta}(x_j, x_j, \omega) \right] \right\} .$$

(24)

The term $I_j^{(1)}$ corresponds to the limit of ideal non-invasive contacts considered in [20]. It is derived by dropping the second term in equation (21) and the ensuing term in the lesser counterpart. This leads to the exact solution $O(w_j^4)$ of equations (15) and (16) for $\lambda_B = 0$. In the second-order solutions (21) we have introduced the additional approximation of neglecting vertex corrections $\propto w_j^2$ and $\propto \lambda_B$ in the evaluation of the many-body self-energy $\Sigma^{\text{int}}$. Notice that the two probes are completely uncorrelated within the ‘non-invasive’ component $I_j^{(1)}$. In the higher order contribution $I_j^{(2)}$ it is possible to distinguish two kinds of terms. On one hand, those $\propto w_j^4$ account for the effect of dephasing and resistive behavior induced by the inelastic scattering processes due to the coupling to the reservoirs. On the other hand, terms $\propto w_j^2 w_j^2$ describe interference effects between the two probes.

// Additional content relating to the theoretical framework and implications for systems of interest

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It is now convenient to express the lesser and greater Green functions in terms of spectral functions:

\[
G^{\text{Latt}}_{\alpha,\alpha}(x_1 - x_2; \omega + \mu_\alpha) = \lambda_{\alpha}^{<,\omega}(\omega) \rho_\alpha(x_1 - x_2; \omega), \\
G^{\text{Latt}}_{\beta,\beta}(\omega) = \lambda_{\beta/\gamma}^{<,\omega}(\omega) \rho_\beta(\omega),
\]

with \(\lambda_{\alpha}^{<,\omega}(\omega) = i n_{F,\alpha}(\omega), \lambda_{\beta/\gamma}^{<,\omega}(\omega) = -i[1 - n_{F,\beta}(\omega)]\), \(n_{F,\alpha}(\omega) = 1/(e^{(\omega+\mu)/T} + 1)\) being the Fermi function where the upper and lower signs correspond, respectively, to the right and left movers of the wire, and \(n_{F,\beta}(\omega) = 1/(e^{(\omega-\mu)/T} + 1)\), \(\mu_\beta\) being the chemical potentials of the electrons in the \(\beta\) reservoir, relative to the mean chemical potential \(\mu\) of the wire. \(T\) is the temperature, which we assume to be the same for the wire and the probes, while \(\rho_\alpha(\omega) = iG_{\alpha\alpha}^{\text{Latt}}(x_1 - x_2; \omega) - i[G_{\alpha\alpha}^{\text{Latt}}(x_2 - x_1; \omega)]^\dagger\) is the spectral function for the \(\alpha\) movers in the Luttinger model, and \(\rho_\beta(\omega) = -2 \text{Im}[\lambda_{\beta}^{<,\omega}(\omega)]\) is the spectral density of the \(\beta\) probe. Replacing in equations (23) and (24), the full expression for the current reads

\[
\begin{align*}
I_j &= 2w_j^2 \sum_{\alpha,\beta=l,r} \int_\omega^{\infty} \frac{d\omega}{2\pi} \left[ n_{F,\alpha}(\omega) - n_{F,\beta}(\omega) \right] \rho_\beta(\omega) \\
&\times \left[ \rho_\alpha(0, \omega)[1 + 2w_j^2 \text{Re}(G^{\text{Latt}}_{\beta\beta}(\omega)) \text{Re}(G^{\text{Latt}}_{\alpha\beta}(x_j - x_\alpha; \omega)) \right] \\
&+ 2w_\alpha^2 \text{Re}[\rho_\alpha(x_j - x_\alpha; \omega) (G^{\text{Latt}}_{\alpha\alpha}(x_j - x_\alpha; \omega))^*] \\
&+ 2w_\beta^2 \text{Re}[\rho_\beta(x_j - x_\beta; \omega) G^{\text{Latt}}_{\beta\beta}(x_j - x_\beta; \omega)(G^{\text{Latt}}_{\alpha\beta}(x_j - x_\alpha; \omega))] \\
&+ 2w_j^2 \sum_{\alpha,\beta=l,r} \int_\omega^{\infty} \frac{d\omega}{2\pi} \left[ n_{F,\beta}(\omega - \mu_\beta) - n_{F,\beta}(\omega - \mu_\beta) \right] \\
&\times \rho_\beta(\omega) \rho_\beta(\omega) G^{\text{Latt}}_{\beta\beta}(x_j - x_\beta; \omega) (G^{\text{Latt}}_{\alpha\beta}(x_j - x_\alpha; \omega))^*,
\end{align*}
\]

(26)

where we use the notation \(j, j'\) such that \(j = 2\) and \(j' = 1\).

### 2.4. Voltage drop and four-terminal resistance

The chemical potentials \(\mu_j\) in the expressions of section 2.3 must be set to satisfy the condition of local electrochemical equilibrium between the probes and the wire. This implies vanishing flows \(I_j = 0, j = 1, 2\), with the currents defined in equation (26), and the two chemical potentials \(\mu_j\) must satisfy these constraints. In the case of non-invasive probes, the two probes are completely uncorrelated, and the problem can be reduced to that of the wire coupled to a single probe, which senses the local chemical potential of the wire. Instead, in the present case, we have to solve a system of two nonlinear equations to calculate \(\mu_1\) and \(\mu_2\), from which the voltage drop \(\Delta V = \mu_1 - \mu_2\) between the points \(x_1\) and \(x_2\) of the wire coupled to the two probes can be evaluated. This voltage drop contains not only information on the scattering processes in the wire that are independent of the coupling to the probes, but also on the inelastic scattering processes and interference effects introduced by the probes themselves. The four-terminal resistance can be evaluated from equation (1) and the ratio between the four-terminal and two-terminal resistance \(R_2 = V/I\) results:

\[
\frac{R_{4t}}{R_{2t}} = \frac{\Delta V}{V}.
\]

(27)

### 3. Results without impurity

In this section we show results for the ratio between \(R_{4t}/R_{2t}\) in the case of \(\lambda_B = 0\). It is important to mention that in the limit of non-invasive probes, this ratio vanishes identically in this case, and all the features in the behavior of the resistance discussed in this section are solely due to the invasive nature of the probes.

We characterize the strength of e–e interactions with the parameter \(K = (1 + \frac{2e}{\pi})^{1/2}\). The limit of non-interacting electrons corresponds to \(K = 1\) while typical values of \(K\) (experimentally determined in transport measurements in nanodevices [29]) are in the range 0.25 < \(K\) < 0.75.

Results for \(R_{4t}/R_{2t}\) as a function of the bias voltage \(V\) for the probes connected at two fixed positions and different values of the e–e interaction \(K\) are shown in figure 2. In order to gain insight into the behavior of the ratio between the resistances, let us notice that for vanishing bias voltage \(V\), the voltage drop \(\Delta V\) and thus \(R_{4t}/R_{2t}\) should also be vanishing. It is, therefore, not surprising that for low enough \(V\), \(R_{4t}/R_{2t}\) displays a power law behavior as a function of \(V\):

\[
\frac{R_{4t}}{R_{2t}} \propto V^{2\gamma+1}.
\]

(28)

The exponent \(\gamma\) is related to the Luttinger parameter as \(\gamma = (K + K^{-1} - 2)/4\). The latter result can be rather straightforwardly derived from an expansion of \(I_j\) for low \(V\). On the other hand, a classical Ohmic-like resistive behavior implies a constant value of \(R_{4t}/R_{2t}\). In figure 2, it can be seen that such a behavior is approximately attained when the bias voltage overcomes a value \(V_c\), which satisfies \(eV_c \approx \hbar v/(2K(x_1 - x_2))\). This energy scale can be understood by

\[3\] Although we have set \(e = h = \nu = 1\) in this work, we restore these constants in this section in order to express the units of the different quantities here introduced in a more transparent way.
so defined depends on the e–e interaction strength $g$ and the voltage profile of non-invasive probes in a system with an impurity, which is observed in both non-interacting [10, 11, 13] and interacting systems [20]. In those cases the origin is the occurrence of interference in the electronic wavepacket generated by the back-scattering processes that take place at the impurity. In the present case, the interference is originated by scattering processes at the probes. Unlike the behavior for non-invasive probes, in our case the voltage drop induced by the probes has the same sign as the applied external voltage. This means that the four-terminal resistance for invasive probes in a clean wire is always positive, in spite of the presence of back-scattering processes (see [19, 24]). Another interesting observation is that the saturation value $A$ decreases for increasing electron–electron interactions, This indicates that the latter tend to screen the inelastic scattering processes introduced by the coupling to the probes.

In figure 3 we show the behavior of the ratio between resistances with the position of one of the probes kept fixed while the position of the second one is moved along the wire. This pattern reveals that the functional behavior is

$$
\frac{R_{4l}}{R_{2l}} \approx A + B \sin(KV(x_2 - x_1))/V^{2\gamma+1}, \quad V > V_c
$$

(29)

within the high $V > V_c$ voltage regime, corresponding, respectively, to the solid and dashed lines in the figure. The $2k_F$ modulation resembles the behavior found in the voltage profile of non-invasive probes in a system with an impurity, which is observed in both non-interacting [10, 11, 13] and interacting systems [20]. In those cases the origin is the occurrence of interference in the electronic wavepacket generated by the back-scattering processes that take place at the impurity. In the present case, the interference is originated by scattering processes at the probes. Unlike the behavior for non-invasive probes, in our case the voltage drop induced by the probes has the same sign as the applied external voltage. This means that the four-terminal resistance for invasive probes in a clean wire is always positive, in spite of the Friedel-like $2k_F$ oscillations. This is in strong contrast to the case of non-invasive probes, where these oscillations provide a mechanism for $R_{4l} < 0$. Figure 4 illustrates the same situation but for fixed voltage and varying $K$. One sees that, in general, larger values of the e–e interactions produce smaller values of $R_{4l}/R_{2l}$. Then we conclude that, although one cannot have...
negative values of the four-terminal resistance in the absence of impurities, e–e interactions tend to facilitate that possibility.

In figure 5 we show the effect of the temperature on the behavior of $R_{4t}/R_{2t}$. It is clear that, as the temperature increases, the oscillations discussed in figure 2 within the high voltage regime tend to be wiped out and the resistance evolves to a constant value. This behavior is depicted in more detail in figure 6, where we display $R_{4t}/R_{2t}$ as function of $T$ for three different values of the bias voltage $V$. In analogy with the previously discussed behavior found at $T = 0$, as function of $V$ (figure 2), there is a crossover temperature $T_c \approx \hbar v/(2K|x_1 - x_2|)$ which allows low and high temperature regimes to be distinguished. For low temperatures ($T < T_c$), we have verified that the ratio between the resistances behaves as

$$R_{4t}/R_{2t} \approx a + bT^{2\gamma+1},$$

where $a$ and $b$ depend on $V$. For high temperatures $R_{4t}/R_{2t}$ tends to a constant value. As the temperature increases, coherence tends to disappear. For this reason, no signature of the oscillatory behavior observed in figure 2 is found here. The interplay between $T$ and $V$ gives rise to the possible occurrence of a global maximum of the $R_{4t}/R_{2t}$. The value of temperature that corresponds to this maximum, when it is present, depends on $K$ and $|x_1 - x_2|$.

4. Results with impurity

In this section we analyze the behavior of $R_{4t}/R_{2t}$ at $T = 0$, for a wire with an impurity with back-scattering strength $\lambda_B$. In the case of non-invasive probes, the local voltage displays $2k_F$ Friedel-like oscillations with constant amplitude for non-interacting electrons [10, 11, 13], and with modulated amplitude in the case of an interacting wire [20].

Figure 7 shows $R_{4t}/R_{2t}$ for the probes connected at fixed positions, as a function of the position of the impurity $x_0$. Friedel-like oscillations with period $2k_F$ are identified, with an increasing amplitude for increasing back-scattering strength. As in the case of non-invasive probes, the amplitude is modulated for interacting electrons, the local voltage achieving the highest amplitudes at the position of the impurity. Unlike the case of non-invasive probes, the oscillations take place with respect to a constant non-vanishing value, which is determined by the degree of coupling of the probes. For the parameters shown in the figure, $R_{4t}$ is always a positive quantity.

Besides interference effects, it is clear that the coupling of the probes generates classical resistive behavior through inelastic scattering processes, while the elastic scattering induced by the impurity induces Friedel oscillations. The first type of process takes place with a strength $\propto w_f^2/\lambda_f$, where $\lambda_f$ is the bandwidth of the reservoirs, while the second one takes place with a strength $\lambda_B$. The two mechanisms are competitive regarding the possibility of having $R_{4t} < 0$. In figure 8 we analyze, precisely, this possibility. To this end, we have fixed the first probe at the position $x_1 = x_0 = -10$, where the minimum $R_{4t}/R_{2t}$ is achieved, considering different positions for the second probe $x_2$. For each of these configurations we then vary the ratio $w_f^2/\lambda_f$, to define $(w_f^2/\lambda_f)_{\text{lim}}$, at which $R_{4t}/R_{2t} = 0$. The corresponding results are plotted in the figure for different e–e interactions. For $w_f^2/\lambda_B > (w_f^2/\lambda_B)_{\text{lim}}$ the ratio $R_{4t}/R_{2t}$ is positive for any value
of $\lambda_3$. On the other hand, the condition $w^2_1/\lambda_1 < (w^2_j/\lambda_j)_{\text{lim}}$ defines the values of coupling strength for which a negative four-terminal resistance is possible, depending on the position of the impurity.

A very interesting and subtle issue that is also revealed by our analysis concerns the role of e–e interactions in the possible occurrence of a negative four-terminal resistance. Based on the results obtained for non-invasive probes [10, 11, 13, 20], one would expect that the e–e interactions oppose such a possibility, owing to the fact that for stronger interactions (smaller values of $K$) the amplitude of the oscillations coming from the presence of the impurity diminishes. However, in the present case this effect competes with the global ‘upward’ shift coming from the contribution of $I_j^{(2)}$. In other words, as already pointed out in section 3, the weak invasiveness of the probes, which in our formulation is contained in $I_j^{(2)}$, produces a voltage drop that has the same sign as the bias $V$. It turns out that the magnitude of such a shift also depends on $K$, and it decreases for increasing interactions (decreasing $K$), as shown in figures 2 and 6. The combination of these two effects gives rise to the result depicted in figure 8, where one sees that for sufficiently separated probes, e–e interactions facilitate the occurrence of a negative four-terminal resistance.

5. Summary and conclusions

We have analyzed the behavior of the four-terminal resistance in a biased quantum wire with an impurity. We have modeled the wire by an infinite-length Luttinger wire where the bias voltage is represented by different chemical potentials for the left and right movers, and the impurity by a back-scattering term. We have also introduced models for the probes, which consist of reservoirs of non-interacting electrons. These systems are locally weakly coupled to the wire and have chemical potentials satisfying the conditions of vanishing electronic currents between the reservoirs and the wire. The difference between the thus determined chemical potentials defines the voltage drop, from which the ratio between the four-terminal and two-terminal resistance can be calculated. We have solved the problem within perturbation theory in the impurity strength and the tunneling parameter defining the coupling between the probes and the wire, within the framework of the non-equilibrium Green functions formalism. We have neglected vertex corrections in the self-energy for the e–e interaction associated with the inelastic scattering processes due to the escape to the leads and the elastic scattering processes at the impurity. Since we have assumed that these two parameters are small enough, the latter is expected to be a reliable approximation.

We have analyzed the voltage drop beyond the non-invasive assumption for the coupling of the probes to the wire. That is, we have studied not only the voltage drop originated by elastic scattering processes along the wire, but also the effects introduced by the coupling to the probes. We have shown that the inelastic scattering processes due to the invasive coupling of the probes induce a voltage drop with a power law behavior as a function of the bias voltage for low values of this parameter, with an exponent determined by the e–e interaction. In the limit of non-interacting electrons, this reduces to a linear dependence as a function of the bias voltage. This behavior has classical and quantum features, since the voltage drop is always in the same sense as the applied voltage but displays a pattern of oscillations indicating quantum interference between the two probes. These features, are, however, screened as the e–e interaction increases. In our calculations, we have considered an infinite wire. However, the separation between the probes sets a natural length scale in the problem, which determines the crossover value of the bias voltage for which inelastic scattering processes become active. In the case of an interacting wire of finite length, we expect that our results will remain valid provided the length of the wire is much larger than the separation between the probes. In the presence of an impurity, the elastic backward scattering processes and $2k_F$ oscillations detected by non-invasive probes [20] are superimposed on the inelastic processes introduced by the probes.

Our results have an important outcome in relation to experimental measurements of four-terminal resistance in real systems. That is, for invasive probes, elastic effects like those generated by back-scattering processes by impurities can still lead to a voltage drop that opposes the applied voltage, giving rise to a negative four-terminal resistance. However, the amplitude of these processes must be large enough in order to overcome the classical resistive effect introduced by the probes.

As far as the e–e interaction effects are concerned, they play a fundamental role in the calculated magnitudes. For stronger e–e interactions, the oscillation amplitude coming from the impurity decreases. The amplitude of the global shift coming from the interaction of the probes also decreases for stronger interactions. We have shown that if the separation of the probes is large enough, the possibility of measuring a negative resistance increases for stronger interactions.

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Appendix. Green functions and spectral functions for the Luttinger wire and the reservoirs

We can follow the procedure of [30] to evaluate the retarded Green functions of the Luttinger wire and calculate the spectral density from \( \rho_0(x, x'; \omega) = i[G_R^0(x - x'; \omega) - G_D^0(x - x'; \omega)] \). The result is

\[
\rho_0(x, x'; \omega + i\epsilon) = C \exp \left[ \mp \left( \frac{i}{\epsilon} - k_F \right) x \right] \times |\omega|^2 \phi(y, 2\gamma + 1, \pm 2\nu \omega/v), \tag{A.1}
\]

where \( \phi \) is Kummer’s hypergeometric function, \( y = (K + K^{-1} - 2)/4 \) and \( v = 1/K \). In order to perform numerical calculations, we introduce an energy cutoff \( \Lambda \) such that \( \rho(x, \omega + \mu) \to \rho(x, \omega + \mu)\Theta(\Lambda - |\omega|) \). C is a normalization constant, which can be evaluated by the normalization condition

\[
\int \frac{d\omega}{2\pi} \rho(x, \omega)|_{\omega=0} = 1. \tag{A.2}
\]

The retarded and advanced Green functions are then calculated using the Kramers–Kronig relations

\[
G^R_{\text{Lutt}}(x, \omega) = \int \frac{d\omega'}{2\pi} \frac{\rho(x, \omega')}{\omega - \omega' + i\eta} \tag{A.3}
\]

\[
= \frac{1}{2\pi} \text{Re} \left[ \rho(x, \omega') \right] + \text{Im} \left[ \rho(x, \omega') \right] \tag{A.4}
\]

\[
P \int \frac{d\omega'}{2\pi} \rho(x, \omega') = \frac{1}{\rho}(x, \omega). \tag{A.5}
\]

The real part is evaluated numerically by using the procedure explained in [31]. We have verified that with a cutoff \( \Lambda \approx 20 \) the evaluated voltage drop \( \Delta V \) is independent of this cutoff.

For the reservoirs, we consider a constant density of states \( \pm \Delta' \). Therefore the retarded Green function for the probes can be calculated using the Kramers–Kronig relations and gives

\[
r^R_j(0, \omega + \mu) = \frac{1}{2\Lambda} \ln \frac{|\omega + \Delta'|}{|\omega - \Delta'|} - \frac{i\pi}{2\Lambda} \Theta(\Lambda' - |\omega|). \tag{A.5}
\]

References


