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Dear Jim

Thank you for your queries about the "constant" part of the graph of a function.

1. Let $f:\mathbb{R} \to \mathbb{R}$ be bounded and differentiable in $[a,b]$. Then I claim, but don't know how to prove, that f can be decomposed as follows:

$$f = g + h \ , \qquad , \ g'(x) \neq 0 \text{ for every } x \in [a,b],$$

where g cannot be further decomposed in the same way and h is a constant function. The former I call the 'variable', the latter the 'constant' part of f.

2. In your examples, the constant part of $y = x$ is 0 whereas that of $y = x^2 + 1$ is 1 .

3. I suppose that the best way of detaching the "constant" part of a function is to expand it into Fourier series, the first term of which is always constant. The Taylor series won't do for, as you point out yourself, it gives the origin an unjustified privilege. Besides, the constant term of a Fourier series has a rather natural interpretation: it is (twice) the average of the given function over the interval.

4. The point of decomposing the graph of a function into a constant" and a "variable" part is not a mathematical one but that of distinguishing changes in time from constant (or fairly constant) values of the property represented by the function. If one assumes that only the "variable" part of a function represents a process, then one needs such a separation. This was my case: to define mental functions in neural terms I had to detach the part of the state function of a plastic neural system that varies in the course of time.

Cordially

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